

Ferrara-Porrati-Sagnotti approach and the one-dimensional supersymmetric model with PBGS

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Abstract

We apply Ferrara-Porrati-Sagnotti approach to the case of one-dimensional supersymmetric model with $N = 2$ supersymmetry spontaneously broken to the $N = 1$ one. We explicitly demonstrate that only one superfield can be treated as the Goldstone one, while the second one has the meaning of the matter superfield. The general action for such a system is constructed and also two relevant particular cases are considered.

1 Introduction

In [1] *J. Bagger and A. Galperin (BG)* proposed the approach to construct the $N = 2, D = 4$ supersymmetric Born-Infeld theory. Combining two $N = 1$ supermultiplets, namely, $N = 1$ *vector* and $N = 1$ *chiral* ones, they realized the transformations of additional $N = 1$ supersymmetry with parameters $\eta_\alpha, \bar{\eta}_{\dot{\alpha}}$ as

$$\delta(W)_\alpha = \left(1 - \frac{1}{4}\overline{D^2 X}\right)\eta_\alpha - i\partial_{\alpha\dot{\alpha}}X\bar{\eta}^{\dot{\alpha}}, \quad \delta X = -2(W)^\alpha\eta_\alpha, \quad (1.1)$$

Due to the presence of the constant term in this transformation law, this additional supersymmetry is spontaneously broken. Keeping in mind, that from the vector superfield W_α Bianchi identity, for the $N = 1$ vector supermultiplet it follows that

$$D^2 W_\alpha \sim \partial_{\alpha\dot{\alpha}}\overline{W}^{\dot{\alpha}}, \quad (1.2)$$

then the trivial action

$$S = \int d^4x d^2\theta X \quad (1.3)$$

becomes invariant under transformations (1.1). However, the action (1.3) acquires a real meaning only after introducing the additional constraint (which is invariant under (1.1))

$$W \cdot W + X \left(1 - \frac{1}{4}\overline{D^2 X}\right) = 0. \quad (1.4)$$

Once again, due to the presence of the constant in (1.4), this constraint can be explicitly solved by expressing X in terms of the superfields $W_\alpha, \overline{W}_{\dot{\alpha}}$. The resulting action is just the $N = 2$ Born-Infeld one.

In the recent paper [2] *S. Ferrara, M. Porrati and A. Sagnotti (FPS)* proposed the generalization of the *BG* construction to the cases of several $N = 1$ vector supermultiplets. The *FPS* approach includes *two* basic ingredients.

Firstly, the nonlinear constraint (1.4) is generalized to be

$$d_{abc} \left(W_b \cdot W_c + X_b \left(m_c - \frac{1}{4}\overline{D^2 X}_c \right) \right) = 0, \quad (1.5)$$

where $W_{a\alpha}$ and X_a are n -copies of $N = 1$ vector and chiral multiplets respectively, and the totally symmetric tensor d_{abc} and vector m_a are a set of constants.

The realization of hidden $N = 1$ supersymmetry is similar to the *BG* case and reads

$$\delta W_{a\alpha} = \left(m_a - \frac{1}{4}\overline{D^2 X}_a \right) \eta_\alpha - i\partial_{\alpha\dot{\alpha}}X_a\bar{\eta}^{\dot{\alpha}}, \quad \delta X_a = -2W_a^\alpha\eta_\alpha. \quad (1.6)$$

The invariance of (1.5) with respect to (1.6) involves the new *additional* constraint

$$d_{abc}W_{b\alpha}X_c = 0, \quad (1.7)$$

which was automatically satisfied in the *BG* case. The constraints (1.5), (1.7) can be solved to express the bosonic $N = 1$ chiral superfields X_a through the $N = 1$ vector supermultiplets $W_{a\alpha}$.

The next nontrivial step of the *FPS* approach yields the structure of the corresponding action, which proved to be more involved

$$S = \int d^4x d^2\theta \left[e^a X_a + C_{ab} \left(W_a \cdot W_b + X_a \left(m_b - \frac{1}{4}\overline{D^2 X}_b \right) \right) + c.c. \right], \quad (1.8)$$

where $e^a = \text{const}$, $C_{ab} = C_{ba} = \text{const}$. In fact, the additional term with C_{ab} is invariant with respect to S -transformations (1.6). Such a term does not exist in the case of one supermultiplet, but it proved to be *essential* in the cases of several supermultiplets. The action (1.8) with X_a being the solution of the constraints (1.5), (1.7) is treated as the many-field extension of $N = 2$ Born-Infeld theory.

In [2] the detailed analysis of the $n = 2$ case was presented, which can be divided into two subcases:

- $d_{111} = 1, d_{112} = -1 \quad I_4 = 0,$

- $d_{111} = 1, d_{122} = -1 \quad I_4 > 0$,

where I_4 is a quartic invariant. Then, the analysis of the next $n = 3$ case has been given in [3]. The above, quite short sketch of the *FPS* approach is enough to raise *two* interesting questions:

- whether the supermultiplets with *different* constants d_{abc} are really *different*?
- whether the action (1.8) is *unique* as it happened in the $n = 1$ case?

The reasons for raising the first question are the following.

After resolving the constraints (1.5), (1.7), we will have in the theory n copies of $N = 1$ vector multiplets $W_{a\alpha}$ with highly nonlinear transformation properties under hidden $N = 1$ supersymmetry (1.6). The presence of the constants m_a in (1.6) at first sight means that all of them have Goldstone nature. But it cannot be so, because the spontaneous breaking of $N = 2$ to $N = 1$ supersymmetries is accompanied by only one Goldstone superfield [4, 5]. Therefore, it should exist a special basis in which only one superfield is a Goldstone one, while the remaining superfields have to be matter superfields with trivial transformation properties with respect to the broken S -supersymmetry. Due to close relations between linear and nonlinear realizations of partially broken supersymmetries [6], the same property has to be visible within the linear realizations method, the *FPS* approach is dealing with. Thus, the claim that different choices of the constants d_{abc} in the basic constraints (1.5), (1.7) must be equivalent, should be carefully analyzed.

The second question about the uniqueness of the action we are raising, is also related with the existence of the special basis in which we have one Goldstone superfield and an arbitrary number of matter superfields. In such a basis the general action will have a functional freedom. Thus, it has to be quite interesting to analyze this general action and understand the reasons that just select the *FPS* actions.

Unfortunately, the analysis of $N = 2$ extended Born-Infeld theory is quite involved, and the ideological part is hidden behind complicated calculations. Fortunately enough, the *FPS* approach, as we demonstrated in [7], is not limited by application to the Born-Infeld theory only, and the questions we raised above may be answered through the analysis of a much simpler system, which the *FPS* approach can be applied to.

In our paper [8] we explored the generalization of the *FPS* construction for the simplest model of super-particle which realized the partial breaking of $N = 2$ to $N = 1$ supersymmetry in one dimension. We explicitly established the transformations that relate two different $n = 2$ cases with the special basis discussed above, as well as the relations between them. We also constructed the general action for such a system containing two arbitrary functions and provided the explicit form of these functions reducing the general action to *FPS*-like ones.

In this contribution we review our results.

2 Partial breaking in one dimension and the superparticle action

We begin with a simple example of a superparticle model describing $N = 2 \rightarrow N = 1$ partial breaking of global supersymmetry in $d = 1$, which was considered in paper [9] The anticommutation relation of $N = 1, d = 1$ Poincaré superalgebra reads

$$\{Q, Q\} = 2P, \quad (2.1)$$

and the coordinates (t, θ) of $N = 1$ superspace satisfy the following rules of complex conjugation: $t^\dagger = -t, \theta^\dagger = \theta$. Now we define the bosonic and fermionic superfields $v(t, \theta)$ and $\psi(t, \theta)$ related as

$$\psi = \frac{1}{2} Dv, \quad (v^\dagger = -v, \psi^\dagger = \psi), \quad (2.2)$$

where the spinor derivative D satisfies the relation

$$D = \frac{\partial}{\partial \theta} + \theta \partial_t, \quad \{D, D\} = 2\partial_t. \quad (2.3)$$

2.1 Linear realization: one-particle case

In full analogy with *BG* approach [1], we introduce two spinor superfields $\psi(t, \theta)$ and $\nu(t, \theta)$, assuming that an additional spontaneously broken $N = 1$ supersymmetry is realized on them as

$$\delta\psi = \varepsilon(1 - D\nu), \quad \delta\nu = \varepsilon D\psi. \quad (2.4)$$

The presence of the constant shift in the first relation suggests the interpretation of $\psi(t, \theta)$ as the Goldstone fermion accompanying the $N = 2 \rightarrow N = 1$ breaking. The superfield $\nu(t, \theta)$, due to its transformation properties under the broken $N = 1$ supersymmetry, may be chosen as a *Lagrangian density*, since the integral over the superspace

$$S = \int dt d\theta \nu \quad (2.5)$$

is invariant with respect to both broken and unbroken supersymmetries.

To be meaningful, the action (2.5) should be accompanied by an additional constraint, which is invariant under transformations (2.4). One may easily check, that the corresponding invariant constraint looks quite similar to (1.4)

$$\psi D\psi - \nu(1 - D\nu) = 0 \quad (2.6)$$

with evident solution given by

$$\nu = \frac{2\psi D\psi}{1 + \sqrt{1 - 4(D\psi)^2}}. \quad (2.7)$$

It is easy to find that from (2.6) it follows that

$$\psi\nu = 0. \quad (2.8)$$

The bosonic limit of the action (2.5) with (2.7) corresponds to the action of a particle in $d = 1$

$$S_{bos} = \frac{1}{2} \int dt \left(1 - \sqrt{1 - (\dot{v})^2} \right). \quad (2.9)$$

Thus, this simplest system demonstrate a close analogy with the *BG* construction of the supersymmetric Born-Infeld action.

2.2 Linear realization: the multiparticle case

The above considered case can be easily extended to that of an arbitrary number of $N = 1$ superfields strictly following the *FPS* approach. Doing so, we firstly introduce a set of n superfields $\psi_a(t, \theta)$ and $\nu_a(t, \theta)$ transforming under implicit $N = 1$ supersymmetry as

$$\delta\psi_a = \varepsilon(m_a - D\nu_a), \quad \delta\nu_a = \varepsilon D\psi_a, \quad (2.10)$$

where m_a are arbitrary constants. Now, again in full analogy with *FPS* approach, we impose the following generalization of the constraint (2.6)

$$d_{abc}(\psi_b D\psi_c - \nu_b(m_c - D\nu_c)) = 0. \quad (2.11)$$

The invariance of this constraint under the broken S -supersymmetry leads to an additional restriction

$$\partial_t(d_{abc}\psi_b\nu_c) = 0, \quad (2.12)$$

which can be reformulated as

$$d_{abc}\psi_b\nu_c = 0, \quad (2.13)$$

assuming that the constant of integration is equal to zero. Finally, the *FPS*-like generalization of the action reads

$$S = \int dt d\theta \left[e^a \nu_a + C_{ab}(\psi_a D\psi_b - \nu_a(m_b - D\nu_b)) \right], \quad (2.14)$$

where e^a and $C_{ab} = C_{ba}$ are arbitrary real constants.

After repeating the basic steps of the *FPS* approach, let us consider the system of two superparticles in the details.

3 Standard basis for $n = 2$ cases

In the standard approach of the Partial Breaking of Global Supersymmetry, the two-particle model should contain only one Goldstone superfield (with the Goldstone fermion among their components). The rest of the fields must be matter ones, i.e. they should not transform with respect to the broken S-supersymmetry. Now we demonstrate how such a splitting works in a two-particle system for special choices of the components of tensor d_{abc} . We show that there is a special basis in which the systems in question, corresponding to a different choice of d_{abc} , can be described in a unified way.

3.1 The case with $d_{111} = d_{222} = 1$

With such a choice of the symmetric tensor d_{abc} , the basic constraints (2.11), (2.13) have a splitting form

$$\begin{aligned}\psi_1 D\psi_1 - \nu_1 (m_1 - D\nu_1) &= 0, & \psi_1 \nu_1 &= 0, \\ \psi_2 D\psi_2 - \nu_2 (m_2 - D\nu_2) &= 0, & \psi_2 \nu_2 &= 0.\end{aligned}\tag{3.1}$$

By rescaling of the variables ψ_a and ν_a in (3.1), one may always choose

$$m_1 = m_2 = 1,\tag{3.2}$$

then the transformations of fermions ψ_a and ν_a in (2.10) take the form

$$\begin{aligned}\delta\psi_1 &= \epsilon (1 - D\nu_1), & \delta\nu_1 &= \epsilon D\psi_1, \\ \delta\psi_2 &= \epsilon (1 - D\nu_2), & \delta\nu_2 &= \epsilon D\psi_2.\end{aligned}\tag{3.3}$$

Similarly to the one particle case (2.7), the constraints (3.1) can be solved as

$$\nu_1 = \frac{2\psi_1 D\psi_1}{1 + \sqrt{1 - 4(D\psi_1)^2}}, \quad \nu_2 = \frac{2\psi_2 D\psi_2}{1 + \sqrt{1 - 4(D\psi_2)^2}}.\tag{3.4}$$

The FPS-like action (1.8), invariant with respect to both S - and Q -supersymmetries, for this case reads

$$S = \int dt d\theta \left\{ e_1 \nu_1 + e_2 \nu_2 + C_{12} (\psi_1 D\psi_2 - \nu_1 (1 - D\nu_2) + \psi_2 D\psi_1 - \nu_2 (1 - D\nu_1)) \right\}.\tag{3.5}$$

Now we introduce two Goldstone spinor superfields ξ_a that transform as

$$\delta\xi_1 = \epsilon + \epsilon \xi_1 \partial_t \xi_1, \quad \delta\xi_2 = \epsilon + \epsilon \xi_2 \partial_t \xi_2.\tag{3.6}$$

It is easy to check that the *tilded fields* $\tilde{\psi}_a$ and $\tilde{\nu}_a$

$$\tilde{\psi}_a = \psi_a - \xi_a (1 - D\nu_a), \quad \tilde{\nu}_a = \nu_a - \xi_a D\psi_a,\tag{3.7}$$

transform under (3.3), (3.6) as

$$\delta\tilde{\psi}_a = \epsilon \xi_a \partial_t \tilde{\psi}_a, \quad \delta\tilde{\nu}_a = \epsilon \xi_a \partial_t \tilde{\nu}_a.\tag{3.8}$$

Thus, it is a covariant operation to put the superfields (3.7) equal to zero, which is equivalent to the relations

$$\psi_a - \xi_a (1 - D\nu_a) = 0, \quad \nu_a - \xi_a D\psi_a = 0.\tag{3.9}$$

Moreover, the constraints $\tilde{\psi}_a = \tilde{\nu}_a = 0$ contain some additional information: they express the superfields of the linear realization ψ_a and ν_a in terms of the Goldstone superfields ξ_a

$$\nu_a = \frac{\xi_a D\xi_a}{1 + D\xi_a D\xi_a}, \quad \psi_a = \frac{\xi_a}{1 + D\xi_a D\xi_a} \quad (\text{no summation over } a).\tag{3.10}$$

Until now we have *two* Goldstone superfields $\xi_a(t, \theta)$, while we are expecting to have *only one* essential Goldstone fermionic superfield and *one matter* superfield. This may be achieved by passing to the *new* superfields $\eta(t, \theta)$ and $\lambda(t, \theta)$

$$\eta = \frac{1}{2} (\xi_1 + \xi_2) + \frac{1}{4} \xi_1 \xi_2 (\dot{\xi}_1 - \dot{\xi}_2), \quad \lambda = \frac{1}{2} (\xi_1 - \xi_2) + \frac{1}{4} \xi_1 \xi_2 (\dot{\xi}_1 + \dot{\xi}_2),\tag{3.11}$$

which, in virtue of (3.6), transform as expected¹

$$\delta\eta = \epsilon + \epsilon\eta\partial_t\eta, \quad \delta\lambda = \epsilon\eta\partial_t\lambda. \quad (3.12)$$

From (3.12) it follows that $\eta(t, \theta)$ is the fermionic Goldstone superfield accompanying the spontaneous breaking of $N = 2$ supersymmetry, while $\lambda(t, \theta)$ is a matter fermionic superfield. Finally, the superfields $\xi_a(t, \theta)$ as well as $\psi_a(t, \theta)$ and $\nu_a(t, \theta)$ can be expressed in terms of $\eta(t, \theta)$ and $\lambda(t, \theta)$:

$$\xi_1 = \eta + \lambda + \eta\lambda(\dot{\eta} + \dot{\lambda}), \quad \xi_2 = \eta - \lambda - \eta\lambda(\dot{\eta} - \dot{\lambda}), \quad (3.13)$$

$$\psi_1 = \frac{\eta + \lambda}{1 + (D\eta + D\lambda)^2} + \eta\lambda(\dot{\eta} + \dot{\lambda}) \frac{1 - (D\eta + D\lambda)^2}{[1 + (D\eta + D\lambda)^2]^2}, \quad \psi_2 = \psi_1|_{\lambda \rightarrow -\lambda}, \quad (3.14)$$

$$\nu_1 = (D\eta + D\lambda) \left(\frac{\eta + \lambda}{1 + (D\eta + D\lambda)^2} + \frac{2\eta\lambda(\dot{\eta} + \dot{\lambda})}{[1 + (D\eta + D\lambda)^2]^2} \right), \quad \nu_2 = \nu_1|_{\lambda \rightarrow -\lambda}.$$

Thus, the system of constraints (3.1) represents a *non-standard* description of the essential Goldstone superfield $\eta(t, \theta)$ and a matter fermionic superfield $\lambda(t, \theta)$.

3.2 The case with $d_{111} = 1, d_{122} = -1$

For these values of the constants the constraints (2.11) read

$$\begin{aligned} \psi_1 D\psi_1 - \psi_2 D\psi_2 - \nu_1(m_1 - D\nu_1) + \nu_2(m_2 - D\nu_2) &= 0, \\ \psi_1 D\psi_2 + \psi_2 D\psi_1 - \nu_1(m_2 - D\nu_2) - \nu_2(m_1 - D\nu_1) &= 0, \end{aligned} \quad (3.15)$$

while those in (2.13) take form

$$\psi_1\nu_1 - \psi_2\nu_2 = 0, \quad \psi_1\nu_2 + \psi_2\nu_1 = 0. \quad (3.16)$$

Now we introduce the complex superfields ψ, ν and complex parameter m as

$$\begin{aligned} \psi &= \psi_1 + i\psi_2, & \nu &= \nu_1 + i\nu_2, & m &= m_1 + im_2, \\ \bar{\psi} &= \psi_1 - i\psi_2, & \bar{\nu} &= \nu_1 - i\nu_2, & \bar{m} &= m_1 - im_2, \end{aligned} \quad (3.17)$$

and then rewrite equations (3.15) and (3.16) in the splitting form

$$\begin{aligned} \psi D\psi - \nu(m - D\nu) &= 0, & \psi\nu &= 0, \\ \bar{\psi} D\bar{\psi} - \bar{\nu}(\bar{m} - D\bar{\nu}) &= 0, & \bar{\psi}\bar{\nu} &= 0. \end{aligned} \quad (3.18)$$

The solution to the equations (3.18) read

$$\nu = \frac{2\psi D\psi}{m + \sqrt{m^2 - 4(D\psi)^2}}, \quad \bar{\nu} = \frac{2\bar{\psi} D\bar{\psi}}{\bar{m} + \sqrt{\bar{m}^2 - 4(D\bar{\psi})^2}}. \quad (3.19)$$

In terms of these variables the invariant *FPS* action (1.8) acquires a form

$$S = \int dt d\theta \left\{ e\nu + e^*\bar{\nu} + C_{12}(\psi D\bar{\psi} - \nu(\bar{m} - D\bar{\nu}) + \bar{\psi} D\psi - \bar{\nu}(m - D\nu)) \right\}, \quad (3.20)$$

where $e = \frac{1}{2}(e_1 - ie_2)$. Let us note, that one may always choose the constants m_1, m_2 as [7]

$$m_1 = 1, m_2 = 0, \quad \Rightarrow \quad m = \bar{m} = 1. \quad (3.21)$$

With this choice of the parameter m , the complex superfields ψ and ν transform with respect to the broken S -supersymmetry as

$$\begin{aligned} \delta\psi &= \epsilon(1 - D\nu), & \delta\nu &= \epsilon D\psi, \\ \delta\bar{\psi} &= \epsilon(1 - D\bar{\nu}), & \delta\bar{\nu} &= \epsilon D\bar{\psi}, \end{aligned} \quad (3.22)$$

¹This is the form-variation of the fields under implicit $N = 1$ supersymmetry: $\delta\mathcal{A} = \mathcal{A}'(t, \theta) - \mathcal{A}(t, \theta)$.

and obey the constraints

$$\begin{aligned}\psi D\psi - \nu(1 - D\nu) &= 0, & \psi \nu &= 0, \\ \bar{\psi} D\bar{\psi} - \bar{\nu}(1 - D\bar{\nu}) &= 0, & \bar{\psi} \bar{\nu} &= 0.\end{aligned}\tag{3.23}$$

So, we have a full analogy with the previously considered splitting case.

Let us introduce two Goldstone spinor superfields $\xi(t, \theta)$ and $\bar{\xi}(t, \theta)$ which transform as

$$\delta\xi = \epsilon + \epsilon\xi\partial_t\xi, \quad \delta\bar{\xi} = \epsilon + \epsilon\bar{\xi}\partial_t\bar{\xi}.\tag{3.24}$$

Then the *tilded fields* $\tilde{\psi}, \tilde{\bar{\psi}}$ and $\tilde{\nu}, \tilde{\bar{\nu}}$, which are defined by the following expressions

$$\tilde{\psi} = \psi - \xi(1 - D\nu), \quad \tilde{\nu} = \nu - \xi D\psi, \quad \tilde{\bar{\psi}} = \bar{\psi} - \bar{\xi}(1 - D\bar{\nu}), \quad \tilde{\bar{\nu}} = \bar{\nu} - \bar{\xi} D\bar{\psi},\tag{3.25}$$

transform under (3.22), (3.24) as

$$\delta\tilde{\psi} = \epsilon\xi\partial_t\tilde{\psi}, \quad \delta\tilde{\nu} = \epsilon\xi\partial_t\tilde{\nu}, \quad \delta\tilde{\bar{\psi}} = \epsilon\bar{\xi}\partial_t\tilde{\bar{\psi}}, \quad \delta\tilde{\bar{\nu}} = \epsilon\bar{\xi}\partial_t\tilde{\bar{\nu}}.\tag{3.26}$$

Thus, one may again to put these superfields equal to zero, that means

$$\psi - \xi(1 - D\nu) = 0, \quad \bar{\psi} - \bar{\xi}(1 - D\bar{\nu}) = 0, \quad \nu - \xi D\psi = 0, \quad \bar{\nu} - \bar{\xi} D\bar{\psi} = 0.\tag{3.27}$$

It is obvious that the constraints (3.23) follow from (3.27). The constraints (3.27) can be solved as

$$\nu = \frac{\xi D\xi}{1 + D\xi D\xi}, \quad \psi = \frac{\xi}{1 + D\xi D\xi}, \quad \bar{\nu} = \frac{\bar{\xi} D\bar{\xi}}{1 + D\bar{\xi} D\bar{\xi}}, \quad \bar{\psi} = \frac{\bar{\xi}}{1 + D\bar{\xi} D\bar{\xi}}.\tag{3.28}$$

In full analogy with the previously considered case, we introduce the superfields $\eta(t, \theta)$ and $\lambda(t, \theta)$ by

$$\eta = \frac{1}{2}(\xi + \bar{\xi}) + \frac{1}{4}\xi\bar{\xi}(\dot{\xi} - \dot{\bar{\xi}}), \quad \lambda = \frac{1}{2}(\xi - \bar{\xi}) + \frac{1}{4}\xi\bar{\xi}(\dot{\xi} + \dot{\bar{\xi}}),\tag{3.29}$$

and find that they have the same transformation properties as in (3.12)

$$\delta\eta = \epsilon + \epsilon\eta\partial_t\eta, \quad \delta\lambda = \epsilon\eta\partial_t\lambda.\tag{3.30}$$

The superfields $\xi(t, \theta), \bar{\xi}(t, \theta)$ as well as the genuine superfields $\psi(t, \theta), \nu(t, \theta)$ can be expressed in terms of the Goldstone superfield $\eta(t, \theta)$ and matter fermionic superfield $\lambda(t, \theta)$:

$$\xi = \eta - i\lambda - i\eta\lambda(\dot{\eta} - i\dot{\lambda}), \quad \bar{\xi} = \eta + i\lambda + i\eta\lambda(\dot{\eta} + i\dot{\lambda}),\tag{3.31}$$

$$\psi = \frac{\eta - i\lambda}{1 + (D\eta - iD\lambda)^2} - i\eta\lambda(\dot{\eta} - i\dot{\lambda}) \frac{1 - (D\eta - iD\lambda)^2}{[1 + (D\eta - iD\lambda)^2]^2}, \quad \bar{\psi} = (\psi)^\dagger,\tag{3.32}$$

$$\nu = (D\eta - iD\lambda) \left(\frac{\eta - i\lambda}{1 + (D\eta - iD\lambda)^2} - i \frac{2\eta\lambda(\dot{\eta} - i\dot{\lambda})}{[1 + (D\eta - iD\lambda)^2]^2} \right), \quad \bar{\nu} = (\nu)^\dagger.$$

3.3 Relations between the two cases

Since the relations between the pairs of superfields (ξ_1, ξ_2) and (η, λ) as well as between $(\xi, \bar{\xi})$ and (η, λ) are invertible, one can express the spinor fields (ξ_1, ξ_2) through $(\xi, \bar{\xi})$ and vice versa

$$\begin{aligned}\xi &= \frac{1}{2}(1 - i)\xi_1 + \frac{1}{2}(1 + i)\xi_2 + \frac{1}{2}\xi_1\xi_2(\dot{\xi}_1 - \dot{\xi}_2), \\ \bar{\xi} &= \frac{1}{2}(1 + i)\xi_1 + \frac{1}{2}(1 - i)\xi_2 + \frac{1}{2}\xi_1\xi_2(\dot{\xi}_1 + \dot{\xi}_2),\end{aligned}\tag{3.33}$$

$$\begin{aligned}\xi_1 &= \frac{1}{2}(1 + i)\xi + \frac{1}{2}(1 - i)\bar{\xi} + \frac{1}{2}\xi\bar{\xi}(\dot{\xi} - \dot{\bar{\xi}}), \\ \xi_2 &= \frac{1}{2}(1 - i)\xi + \frac{1}{2}(1 + i)\bar{\xi} + \frac{1}{2}\xi\bar{\xi}(\dot{\xi} + \dot{\bar{\xi}}).\end{aligned}\tag{3.34}$$

Thus, at least in one dimension, the two cases are completely equivalent, because they are related by invertible fields redefinitions (3.33), (3.34). The only difference between these cases lies in the diverse definition of the invariant actions (3.5), (3.20). In fact, these two actions are not the most general ones. We will demonstrate this by constructing the most general action for the Goldstone fermion $\eta(t, \theta)$ and the matter fermionic superfield $\lambda(t, \theta)$.

4 Nonlinear realizations approach

In this section we provide an alternative description of the system with a partial breaking of the global $N = 2, d = 1$ supersymmetry within the nonlinear realizations approach. It turns out that in the present case this approach is more suitable for the construction of the most general superfield action.

4.1 NLR approach in one dimension

We start with the $N = 2, d = 1$ Poincaré superalgebra with one central charge generator Z

$$\{Q, Q\} = 2P, \quad \{S, S\} = 2P, \quad \{Q, S\} = 2Z. \quad (4.1)$$

Introducing a coset element g as

$$g = e^{tP} e^{\theta Q} e^{qZ} e^{\eta S}, \quad (4.2)$$

and calculating the expression $g^{-1}dg = \omega_P P + \omega_Q Q + \omega_Z Z + \omega_S S$, one finds the explicit structure for the Cartan forms

$$\omega_P = dt - d\theta\theta - d\eta\eta, \quad \omega_Q = d\theta, \quad \omega_Z = dq - 2d\theta\eta, \quad \omega_S = d\eta. \quad (4.3)$$

The covariant derivatives can be found in a standard way, and now they read

$$\nabla_\theta = D + \eta D\eta \nabla_t, \quad \nabla_t = E^{-1} \partial_t, \quad (4.4)$$

where the explicit forms of the einbein E and its inverse E^{-1} are

$$E = 1 + \eta \partial_t \eta, \quad E^{-1} = 1 - \eta \nabla_t \eta. \quad (4.5)$$

These derivatives satisfy the following (anti)commutation relations

$$\begin{aligned} \{\nabla_\theta, \nabla_\theta\} &= 2(1 + \nabla_\theta \eta \nabla_\theta \eta) \nabla_t, \\ [\nabla_t, \nabla_\theta] &= 2\nabla_t \eta \nabla_\theta \eta \nabla_t. \end{aligned} \quad (4.6)$$

Acting on the coset element g (4.2) from the left by different elements g_0 of the $N = 2, d = 1$ Poincaré supergroup, one can find the transformation properties of the coordinates (t, θ) and the Goldstone superfield $\eta(t, \theta)$. In particular, under both unbroken Q - and broken S -supersymmetries one gets:

- Unbroken supersymmetry ($g_0 = e^{\epsilon Q}$):

$$\delta_Q t = -\epsilon\theta, \quad \delta_Q \theta = \epsilon, \quad (4.7)$$

- Broken supersymmetry ($g_0 = e^{\epsilon S}$):

$$\delta_S t = -\epsilon\eta, \quad \delta_S q = -2\epsilon\theta, \quad \delta_S \eta = \epsilon. \quad (4.8)$$

The final step is to express the Goldstone superfield $\eta(t, \theta)$ in terms of the bosonic superfield $q(t, \theta)$ by imposing the constraint [10]

$$\omega_Z|_{d\theta=0} = 0 \quad \Rightarrow \quad \eta = \frac{1}{2} \nabla_\theta q. \quad (4.9)$$

In order to describe the matter spinor superfield $\lambda(t, \theta)$ within the nonlinear realizations approach, it is necessary to postulate its transformation properties under the S -supersymmetry

$$\delta_S \lambda = 0. \quad (4.10)$$

It is obvious that, due to the transformation of the coordinate t under the S -supersymmetry in (4.8), the variation of $\lambda(t, \theta)$ reads

$$\delta\lambda = \varepsilon\eta\partial_t\lambda. \quad (4.11)$$

One may also define the bosonic superfield $\phi(t, \theta)$ with the transformation property

$$\delta_S\phi = 0, \quad (4.12)$$

which is related with $\lambda(t, \theta)$ in the same way as in (4.9)

$$\lambda = \frac{1}{2}\nabla_\theta\phi. \quad (4.13)$$

4.2 General action

The most general *Ansatz* for the $N = 1$ superfield action describing the Goldstone fermionic superfield $\eta(t, \theta)$ and the matter fermionic superfield $\lambda(t, \theta)$ and having no dimensional constants reads

$$S = \int dt d\theta (1 + \eta\dot{\eta}) \left[\eta F_1^{cov} + \lambda F_2^{cov} + \eta\dot{\eta}\lambda F_3^{cov} + \eta\lambda\nabla_t\lambda F_4^{cov} \right], \quad (4.14)$$

where $F_1^{cov}, F_2^{cov}, F_3^{cov}, F_4^{cov}$ are arbitrary functions depending on $\nabla_\theta\eta$ and $\nabla_\theta\lambda$ only. The reason for such a form of *Ansatz* is quite understandable:

- with respect to S -supersymmetry transformation, $\delta_S t = -\varepsilon\eta$, $\delta_S\eta = \varepsilon$, $\delta_S\lambda = 0$, the “improved” measure $dt d\theta (1 + \eta\dot{\eta})$ is invariant;
- the functions $F_1^{cov}, F_2^{cov}, F_3^{cov}, F_4^{cov}$ as well as $\nabla_t\lambda$ are also invariant with respect to the broken S -supersymmetry;
- the functions $F_1^{cov}, F_2^{cov}, F_3^{cov}, F_4^{cov}$ have to be further restricted by imposing the invariance of the action (4.14) with respect to the broken S -supersymmetry.

The variation of the action (4.14) with respect to S -supersymmetry reads

$$\delta_S S = \int dt d\theta \varepsilon \left[(1 + \eta\dot{\eta}) F_1^{cov} + \dot{\eta}\lambda F_3^{cov} + \lambda\dot{\lambda} F_4^{cov} \right]. \quad (4.15)$$

Thus, the function F_2^{cov} may be chosen to be an arbitrary function of its arguments.

To fix the functions $F_1^{cov}, F_3^{cov}, F_4^{cov}$ from the condition $\delta_S S = 0$ one has, firstly, to integrate over θ in (4.15) and then to replace the arguments of these functions, $\nabla_\theta\eta$ and $\nabla_\theta\lambda$, by

$$\nabla_\theta\eta = (1 + \eta\dot{\eta})D\eta \equiv (1 + \eta\dot{\eta})x, \quad \nabla_\theta\lambda = D\lambda + \eta\dot{\lambda}D\eta \equiv y + \eta\dot{\lambda}x. \quad (4.16)$$

Hence, the explicit expression for each of $F^{cov}[\nabla_\theta\eta, \nabla_\theta\lambda]$ takes the form

$$F^{cov}[\nabla_\theta\eta, \nabla_\theta\lambda] = F[x, y] + \eta\dot{\eta}x(F[x, y])_x + \eta\dot{\lambda}x(F[x, y])_y, \quad (4.17)$$

where, by definition,

$$x \equiv D\eta, \quad y \equiv D\lambda. \quad (4.18)$$

Now

- performing the integration in (4.15) over θ ,
- expanding the functions $F_1^{cov}, F_3^{cov}, F_4^{cov}$ as in (4.17),
- collecting the terms which are linear and cubic in the fermions,
- and assuming that the right hand side of variation is a total derivative with respect to t of an arbitrary function depending on x, y ,

we will get the following three equations

$$\begin{aligned} \left[(1+x^2)F_1 \right]_x - yF_3 &= a, & (a) \\ y \left[(1+x^2)F_1 \right]_{yy} + (y^2F_4)_y &= 0, & (b) \\ (F_3)_y + (F_4)_x &= 0, & (c) \end{aligned} \quad (4.19)$$

where a is an arbitrary constant. The equation (4.19b) may be integrated once, giving

$$y(\tilde{F}_1)_y - \tilde{F}_1 + y^2F_4 = G[x], \quad (4.20)$$

where

$$\tilde{F}_1 \equiv (1+x^2)F_1, \quad (4.21)$$

and $G[x]$ is an arbitrary function depending on x . Then the equation (4.19a) reads

$$(\tilde{F}_1)_x - yF_3 = a. \quad (4.22)$$

Differentiating the equation (4.20) over x and the equation (4.22) over y , and using the equation (4.19c) one may get

$$(G[x])_x = -a \quad \Rightarrow \quad G[x] = -ax - b, \quad (4.23)$$

where b is a new constant. Finally, representing the function \tilde{F}_1 as

$$\tilde{F}_1 = ax + b + y\hat{F}_1 \quad (4.24)$$

we will finish with the equations

$$(\hat{F}_1)_y + F_4 = 0, \quad (\hat{F}_1)_x - F_3 = 0, \quad (4.25)$$

which define the functions F_3, F_4 in terms of an arbitrary function $\hat{F}_1[x, y]$.

Thus, the general action (4.14) is invariant under the broken S -supersymmetry, if the functions F_1, F_3 and F_4 are expressed through arbitrary function F_1 as

$$F_1 = \frac{ax+b}{1+x^2} + \frac{y}{1+x^2}\hat{F}_1, \quad F_3 = (\hat{F}_1)_x, \quad F_4 = -(\hat{F}_1)_y. \quad (4.26)$$

Thus, the most general action for the present system is defined up to two arbitrary functions \hat{F}_1 and F_2 . Let us note that the constant b can be chosen equal to zero, because the action corresponding to the Lagrangian

$$L = \frac{b}{1+x^2} \quad (4.27)$$

is trivial. Thus, the final result for the covariant functions F^{cov} which enter the action (4.14) reads

$$F_1^{cov} = a \frac{\nabla_\theta \eta}{1 + (\nabla_\theta \eta)^2} + \frac{\nabla_\theta \lambda}{1 + (\nabla_\theta \eta)^2} \hat{F}_1^{cov}, \quad F_3^{cov} = (\hat{F}_1)_{\nabla_\theta \eta}, \quad F_4^{cov} = -(\hat{F}_1)_{\nabla_\theta \lambda}, \quad (4.28)$$

with \hat{F}_1^{cov} and F_2^{cov} being arbitrary functions on $\nabla_\theta \eta$ and $\nabla_\theta \lambda$

$$\hat{F}_1 = \hat{F}_1[\nabla_\theta \eta, \nabla_\theta \lambda], \quad F_2 = F_2[\nabla_\theta \eta, \nabla_\theta \lambda]. \quad (4.29)$$

5 Interesting cases

Having at hands the most general expression for the action (4.14), (4.28) invariant with respect to both Q - and S -supersymmetries, it is interesting to visualize the functions F_1, F_2 which reproduce the FPS actions (3.5), (3.20). Comparing (3.14) and (3.32) one may conclude, that these two FPS cases are related through the substitutions: $\lambda \rightarrow -i\lambda, y \rightarrow -iy$ (where $y = D\lambda$). Therefore, it is enough to consider the first case, with $d_{111} = d_{222} = 1$, only.

In order to simplify everything, it is useful to represent the integrand in (4.14) as follows

$$L = \eta F_1 + \lambda F_2 + \lambda \eta \dot{\eta} (x F_2 + \hat{F}_1)_x - \eta \lambda \dot{\lambda} (x F_2 + \hat{F}_1)_y, \quad (5.1)$$

with

$$F_1 \equiv a \frac{x}{1+x^2} + \frac{y}{1+x^2} \hat{F}_1, \quad (5.2)$$

where the functions \hat{F}_1 and F_2 depend on x, y variables (4.18). Comparing the integrands in the actions (3.5) and (5.1), we will get for the two special choices of the constant parameters e_a and C_{ab} of the *FPS* action the following result

$$(a) \quad \text{if } e_2 = C_{ab} = 0, \quad \text{then } L_{FPS} = \nu_1 \Rightarrow \\ F_1 = F_2 = \frac{x+y}{1+(x+y)^2}, \quad a = 1 \quad (5.3)$$

$$(b) \quad \text{if } e_1 = e_2 = C_{ab} = 1, \quad \text{then } L_{FPS} = \psi_1 D\psi_2 + \nu_1 D\nu_2 \Rightarrow \\ F_1 = F_2 = \frac{(x-y)(1+x^2-y^2)}{(1+(x-y)^2)(1+(x+y)^2)}, \quad a = 1. \quad (5.4)$$

Unfortunately, the explicit form of the functions F_1, F_2 which corresponds to *FPS* actions (5.3), (5.4) is not informative enough to understand why these actions were selected.

Let us remind that the general action

$$S = \int dt d\theta (1 + \eta \dot{\eta}) \left[\eta F_1^{cov} + \lambda F_2^{cov} + \eta \dot{\eta} \lambda F_3^{cov} + \eta \lambda \nabla_t \lambda F_4^{cov} \right], \quad (5.5)$$

with the restrictions

$$F_1^{cov} = a \frac{\nabla_\theta \eta}{1 + (\nabla_\theta \eta)^2} + \frac{\nabla_\theta \lambda}{1 + (\nabla_\theta \eta)^2} \hat{F}_1^{cov}, \quad F_3^{cov} = (\hat{F}_1)_{\nabla_\theta \eta}, \quad F_4^{cov} = -(\hat{F}_1)_{\nabla_\theta \lambda} \quad (5.6)$$

contains two arbitrary functions F_1^{cov} and F_2^{cov} . So, as we expected, the invariance with respect to additional, spontaneously broken $N = 1$ supersymmetry does not fix the action in the many-particles case, in contrast with the one-particle case.

6 Conclusion

In this work we analysed in details the application of the *FPS* approach in $d = 1$.

We demonstrated that the resulting system describes the $N = 2, d = 1$ supersymmetric action for two particles in which one of $N = 1$ supersymmetries is spontaneously broken. The final actions possess the same features as the *FPS* ones.

Using the nonlinear realization approach we reconsider the system in the basis where only one superfield has the Goldstone nature while the second superfield can be treated as the matter one. Having at hands the transformations relating the two selected *FPS* cases with our more generic one, we established the field redefinitions which relate these cases. Namely, in this basis, the two *FPS* supermultiplets are related by the redefinition of the matter superfield $\lambda \rightarrow -i\lambda$. This analysis leads to the conclusion that the only difference between two *FPS* cases lies in a various choice of actions, while the supermultiplets specified by the *FPS* constraints are really the same.

Going further on with the nonlinear realization approach, we constructed the most general action for the system of two $N = 1$ superfields possessing one additional hidden spontaneously broken $N = 1$ supersymmetry. This action contains two arbitrary functions and reduces to the *FPS* actions upon specification of these functions.

Unfortunately, the exact form of these functions corresponding to *FPS* actions is not very informative and it gives no hint about the reason why the *FPS* cases were selected. Of course, our consideration was strictly one-dimensional and therefore we cannot argue that all features we discussed will appear in the generalized supersymmetric Born-Infeld theory constructed in [2, 3]:

Nevertheless, we believe that our results and the generality of the nonlinear realization approach are yield reasonable tools to reconsider the questions:

- whether the supermultiplets with different constants d_{abc} are really different?
- which additional properties select the *FPS* actions?

in four dimensions.

Acknowledgments

The work of S.K. was supported by RSCF grant 14-11-00598. The work of A.S was partially supported by RFBR grants 15-02-06670 and 15-52-05022 Arm-a.

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